

# **Failure-Time Mixture Models for Analyzing Time to Response**

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## Time to onset of benefit/response

### Motivation for topic arises from depression

- Current antidepressants take several weeks to obtain a meaningful response
- Fast onset is important unmet need in MDD
  - Patients in the throes of severe symptomatic exacerbation are at risk for days to weeks before experiencing resolution
  - faster onset of action would improve compliance
- This has compelled one to develop drugs with potential for early onset
- How does one assess time to onset and compare 2 treatments
- In depression, there has been much discussion on this topic and no strong consensus on the best analytical approach

In studies of many indications, one collects information longitudinally according to a planned schedule (i.e., weekly, hourly), and if the visits are frequent enough

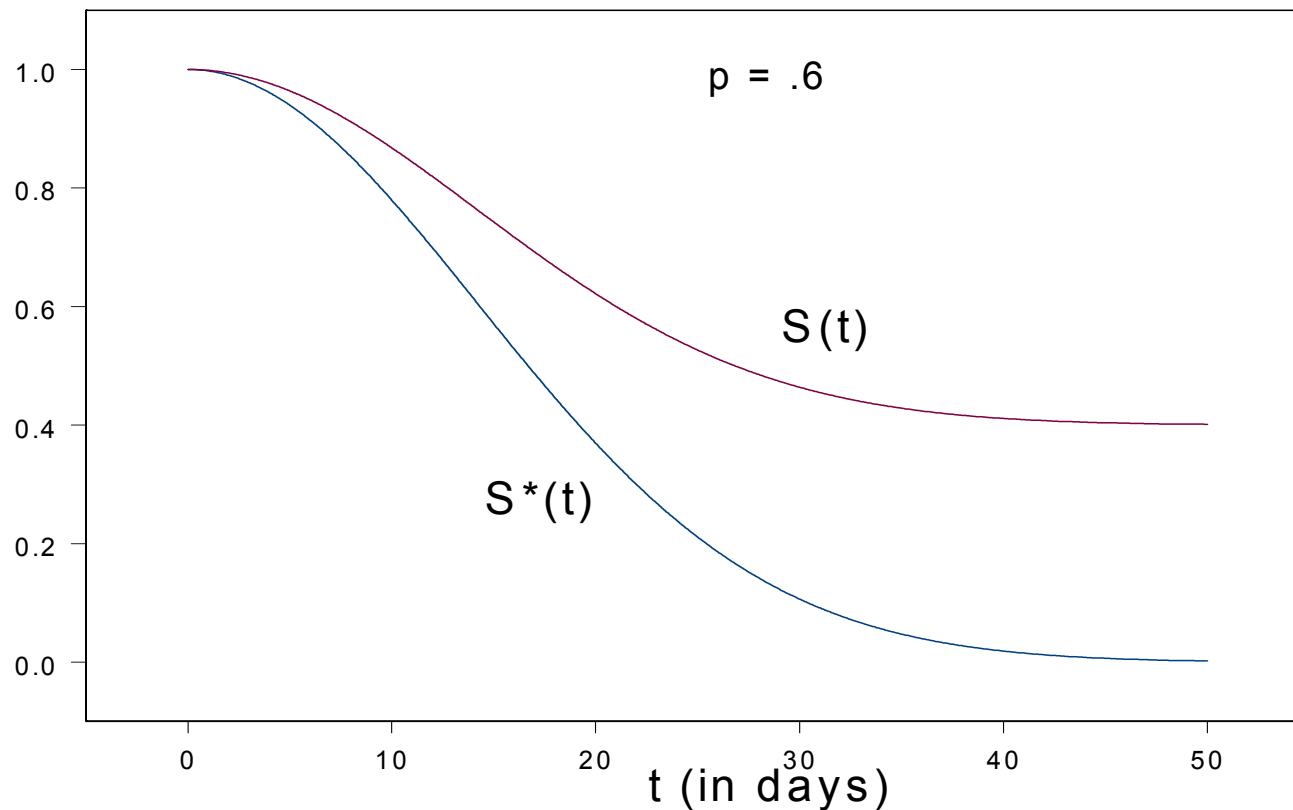
- the onset of action can be assessed by an analysis of the data at early visits
  - i.e., time of onset or response might be considered the time of the first visit when one rejects  $\mu_A = \mu_{PBO}$  OR  $\pi_A = \pi_{PBO}$
  - But difference from pbo need to be sustained at all subsequent visits?
- Many statisticians (Leon, 2001, Mallinckrodt) have recommended the approach above using MMRM (at least in depression)

- Early/Earlier response means **LESS TIME** to response
  - if response can be defined (in terms of a binary variable)
- This compels one to compare the distribution of time to response between/among treatments; i.e. SURVIVAL ANALYSIS METHODS
- But "ordinary" survival analysis is sensitive to both time to response and proportion that respond over a sufficiently long-time period
- If we reject  $H_0 : S_2(t) = S_1(t)$  it is either because proportion that respond are different and/or time to response conditional on response is different. Example from Tamura (2000):
- Trt 1: 80% respond all at week 3. Trt 2: 60% respond all at week 3
- For large sufficiently large N, log-rank would be statistically significant
- But misleading to state trt 1 works faster than trt 2

# Mixture survival models for the analysis of time to response in depression - Laska & Siegel (Psychopharmacology Bulletin, 1995)

- Paradigm: population is a mixture of responders & nonresponders.
- Two properties needed to characterize response to a treatment:
  1.  $p$  - the proportion that respond ( $\geq 50\%$  decrease in B.L HAMD)
  2.  $S^*$  - the distribution of time to response conditional on response
- Try to estimate 1 & 2 for two or more treatments by conducting a RCT
- Moreover for 2 treatments say, want to test:  $p_1 = p_2$  and  $S_1^* = S_2^*$
- In the presence of dropouts, even testing of  $p_1 = p_2$  is not trivial
- Focus of this talk will be on testing  $S_1^* = S_2^*$ 
  - Need to incorporate data from dropouts as this provides information

- For some trt, in a mixed population of responders and nonresponders, denote  $S(t) = \Pr\{T>t\}$
- Denote  $S^*(t) = \Pr\{T>t\}$  among those that will respond to this treatment
- Let  $p$  be the probability of response (i.e., the proportion that respond). It is assumed that those that respond will respond by at least time  $u$
- $$S^*(t) = \Pr\{T>t|T\leq u\} = \frac{P(t < T \leq u)}{P(T \leq u)} = \frac{P(T \leq u) - P(T \leq t)}{P(T \leq u)} = \frac{p - \{1 - S(t)\}}{p} = \frac{S(t) - (1 - p)}{p}$$

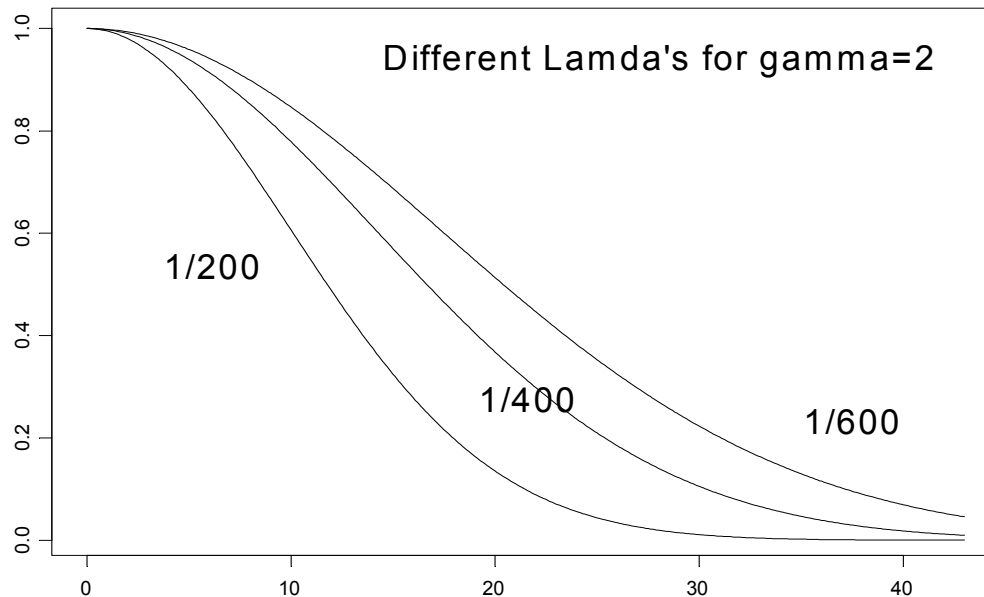


## Estimating $S_1^*$ and $S_2^*$ , and then testing $S_1^* = S_2^*$

- Nonparametric methods (no strong distributional assumptions)
  - use product limit or Kaplan–Meier (KM) methods for estimation
  - to test  $S_1^* = S_2^*$ 
    - ▶ weighted log-rank statistic
    - ▶ Cramer-von Mises statistic
- Parametric mixture methods models
  - Use logistic regression to estimate  $p_1$  and  $p_2$  or a common  $p$
  - Use “time to event” distributions to estimate  $S_1^*$  and  $S_2^*$
  - fit a model with a common  $\hat{S}^*$ , then fit a model resulting in  $\hat{S}_1^*$  and  $\hat{S}_2^*$ —statistically evaluate the change in the likelihood
- Semiparametric mixture models
  - Extension of the Cox P.H. model (Sy and Taylor 2000, Peng and Dear 2000, Li 2010). Will not be covering in this presentation.

# Parametric Mixture Model Approach Using Common Event Time Distributions such as the Weibull or Loglogistic

- Weibull distribution is as central to parametric survival analysis as the normal distribution is to linear models. Some details about the Weibull; for  $t, \lambda, \gamma > 0$
- $S(t) = e^{-\lambda t^\gamma}$      $f(t) = \lambda \gamma t^{\gamma-1} e^{-\lambda t^\gamma}$      $h(t) = \lambda \gamma t^{\gamma-1}$
- $\gamma$  is the shape parameter and  $\lambda$  is the scale parameter; as  $\lambda \uparrow, S^*(t) \downarrow \forall t$





## Consider a Weibull mixture model in the case of two treatments

- Model the probability of response with a logistic regression model

- $\log \frac{p_i}{1-p_i} = \beta_0 + \beta_1 X_i$  where  $p_i = \Pr(Y_i = R)$

$$\Pr(Y = R) = f(y) = \frac{\exp(\beta_0 + \beta_1 X)}{1 + \exp(\beta_0 + \beta_1 X)} \text{ where } X = \begin{cases} 1 & \text{if trt=1} \\ 0 & \text{if trt=2} \end{cases}$$

- For responders, model the time to response with a Weibull

$$f(t|Y = R) = \psi^X \lambda \gamma t^{\gamma-1} e^{-\psi^X \lambda t^\gamma} \text{ where } X = \begin{cases} 1 & \text{if trt=1} \\ 0 & \text{if trt=2} \end{cases}$$

Note that the scale parameter for trt 1 is  $\psi \lambda$ , for trt 2 it is  $\lambda \Rightarrow$  if  $\psi > 1$ ,  
 $S_1^*(t) < S_2^*(t) \forall t$

$$\text{And } S_1^*(t) = e^{-\psi \lambda t^\gamma} = (e^{-\lambda t^\gamma})^\psi = \{S_2^*(t)\}^\psi \Rightarrow \frac{h_1^*(t)}{h_2^*(t)} = \psi$$

- Interpretation: among the responders that have yet to respond, the chance of responding at any particular time, is  $\psi$  times greater in trt 1 than trt 2
- This Weibull model is not only a (conditional) proportional hazards model but it is also a conditional accelerated failure time model
- This means the response time for one treatment is a multiple of the response time for another treatment
- Specifically  $S_1^*(t) = S_2^*(\phi t)$  where  $\phi$  is the acceleration factor
  - ⇒ Proportion of responders that respond by  $t$  in trt 1 is equal to the proportion of responders that respond by  $\phi t$  in trt 2
  - ⇒ Among responders, the time in which the proportion  $\pi$  respond is  $\phi$  times greater in trt 2 than trt 1
- To show that the mixture Weibull model is also an accelerated failure time model, note that
- $S_1^*(t) = S_2^*(\phi t) = e^{-\lambda(\phi t)^\gamma} = e^{-\phi^\gamma \lambda t^\gamma}$
- Recall,  $S_1^*(t) = e^{-\psi \lambda t^\gamma} \Rightarrow \psi = \phi^\gamma$ , or  $\phi = \psi^{1/\gamma}$

- Maximum Likelihood procedures to estimate  $S_1^*$ ,  $S_2^*$  and to test  $S_1^* = S_2^*$

First fit a null model assuming a common  $S^*$  i.e.,  $\theta_{null} = [\beta_0, \beta_1, \gamma, \lambda]'$

Then fit an expanded alternative model allowing a different  $S^*$  for each treatment, i.e.,  $\theta_{alt} = [\beta_0, \beta_1, \gamma, \lambda, \psi]'$ .

Specifically, the construction of the likelihood for estimating  $\theta_{alt}$

- A completing nonresponder's contribution to the likelihood is:

$$\frac{1}{1 + \exp(\beta_0 + \beta_1 X)}$$

- A responder's (who responded at time  $t$ ) contribution to the likelihood is:

$$\frac{\exp(\beta_0 + \beta_1 X)}{1 + \exp(\beta_0 + \beta_1 X)} * \psi^X \lambda \gamma t^{\gamma-1} e^{-\psi^X \lambda t^\gamma}$$

- A nonresponding dropout (at time  $t$ ) contributes this to the likelihood:

$$\frac{1}{1 + \exp(\beta_0 + \beta_1 X)} + \frac{\exp(\beta_0 + \beta_1 X)}{1 + \exp(\beta_0 + \beta_1 X)} * e^{-\psi^X \lambda t^\gamma}$$

- Maximize the log of the joint likelihood with respect to  $\theta_{alt}$  given the data
- No closed form solution: need iterative procedure
- Implement Newton-Raphson with PROC NLP
- Similarly fit the null model, i.e., maximize log of the joint likelihood with respect to  $\theta_{null}$  given the data
- A test of  $S_1^* = S_2^*$  vs  $S_1^* \neq S_2^*$  is based on two times the difference in the maximized log likelihoods which is asymptotically chi-square with 1 d.f.

$$2\{\log f(\hat{\theta}_{alt}|\mathbf{t}, \mathbf{x}) - \log f(\hat{\theta}_{null}|\mathbf{t}, \mathbf{x})\} \sim \chi_1$$

## Mixture Model Using the Loglogistic Distribution

- Model using the Weibull assumes PH, and the hazard is monotonic
- A loglogistic distribution allows for departures from PH
  - Instead, a proportional odds assumption will be imparted
- Loglogistic permits a unimodal hazard
- $S(t) = \frac{1}{1+(t\rho)^\kappa}$      $f(t) = \frac{\kappa t^{\kappa-1} \rho^\kappa}{[1+(t\rho)^\kappa]^2}$      $h(t) = \frac{\kappa t^{\kappa-1} \rho^\kappa}{1+(t\rho)^\kappa}$     for  $t, \rho, \kappa > 0$
- note that as  $\rho \uparrow$ ,  $S^*(t) \downarrow \forall t$

## Consider a loglogistic mixture model in the case of two treatments

- As before, model the probability of response with a logistic regression model

- $\log \frac{p_i}{1-p_i} = \beta_0 + \beta_1 X_i$  where  $p_i = \Pr(Y_i = R)$

$$\Pr(Y = R) = f(y) = \frac{\exp(\beta_0 + \beta_1 X)}{1 + \exp(\beta_0 + \beta_1 X)} \text{ where } X = \begin{cases} 1 & \text{if trt=1} \\ 0 & \text{if trt=2} \end{cases}$$

- For responders, model the time to response with a loglogistic distribution

$$f(t|Y = R) = \frac{\kappa t^{\kappa-1} (\phi^X \rho)^\kappa}{[1 + (t\phi^X \rho)^\kappa]^2} \text{ where } X = \begin{cases} 1 & \text{if trt=1} \\ 0 & \text{if trt=2} \end{cases}$$

Note that "scale" parameter for trt 1 is  $\phi\rho$ , for trt 2 it is  $\rho \Rightarrow$  if  $\phi > 1$ ,  
 $S_1^*(t) < S_2^*(t) \forall t$

- Maximum Likelihood procedures to estimate  $S_1^*$ ,  $S_2^*$  and to test  $S_1^* = S_2^*$

First fit a null model assuming a common  $S^*$  i.e.,  $\theta_{null} = [\beta_0, \beta_1, \kappa, \rho]'$

Then fit an expanded alternative model allowing a different  $S^*$  for each treatment, i.e.,  $\theta_{alt} = [\beta_0, \beta_1, \kappa, \rho, \phi]'$ .

Specifically, the construction of the likelihood for estimating  $\theta_{alt}$

- A completing nonresponder's contribution to the likelihood is:

$$\frac{1}{1 + \exp(\beta_0 + \beta_1 X)}$$

- A responder's (who responded at time  $t$ ) contribution to the likelihood is:

$$\frac{\exp(\beta_0 + \beta_1 X)}{1 + \exp(\beta_0 + \beta_1 X)} * \frac{\kappa t^{\kappa-1} (\phi^X \rho)^\kappa}{[1 + (t\phi^X \rho)^\kappa]^2}$$

- A nonresponding dropout (at time  $t$ ) contributes this to the likelihood:

$$\frac{1}{1 + \exp(\beta_0 + \beta_1 X)} + \frac{\exp(\beta_0 + \beta_1 X)}{1 + \exp(\beta_0 + \beta_1 X)} * \frac{1}{1 + (t\phi^X \rho)^\kappa}$$

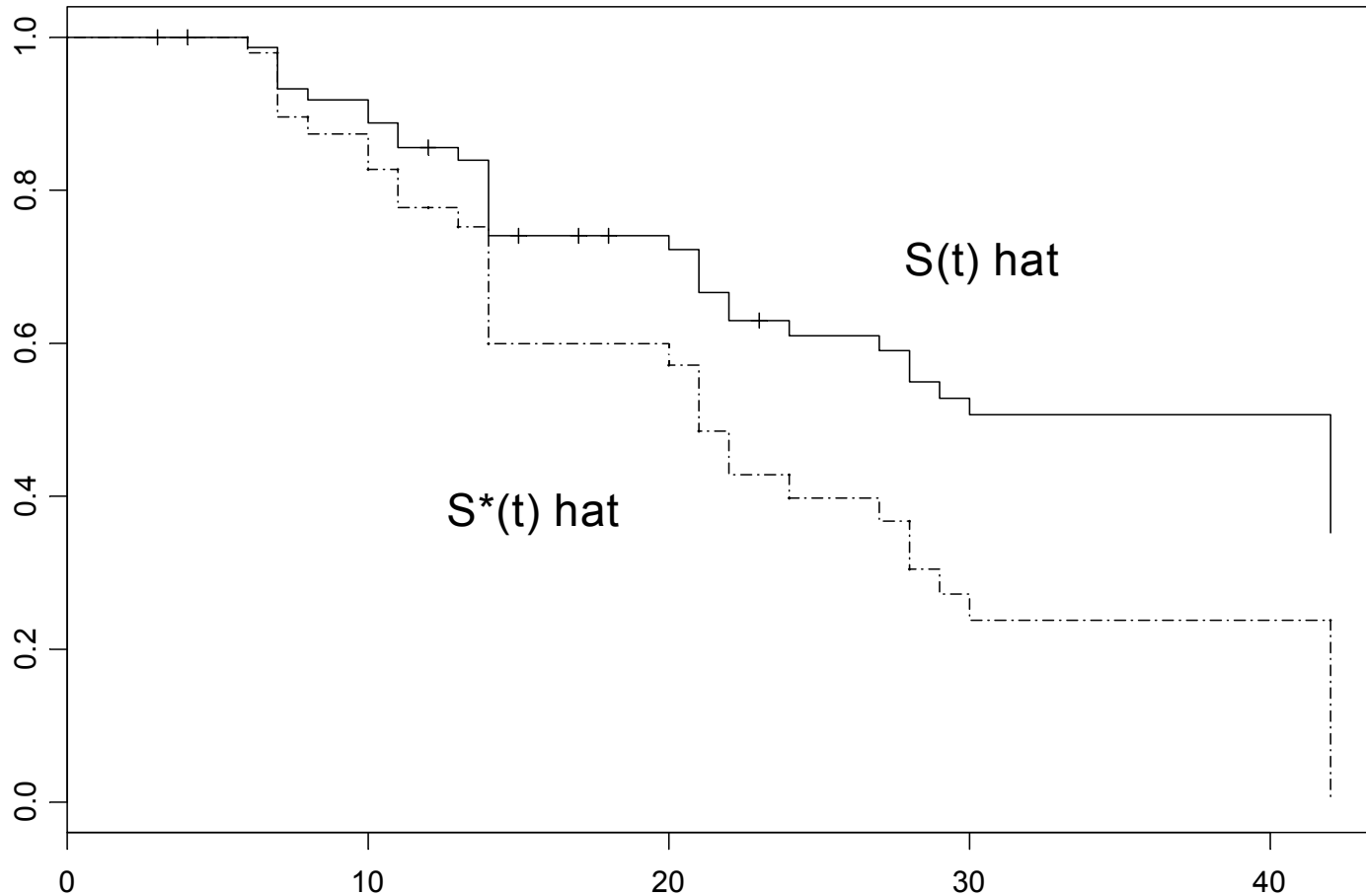
- Since  $S_1^*(t) = \frac{1}{1+(t\phi\rho)^\kappa} = S_2^*(\phi t)$ , this loglogistic model is a conditional accelerated failure time model
  - Interpretation: Among responders, the time in which proportion  $\pi$  response in trt 2 is  $\phi$  times greater than that of trt 1
- To show that the mixture loglogistic model is also a proportional odds model, note that
- $1 - S_1^*(t) = \frac{(t\phi\rho)^\kappa}{1+(t\phi\rho)^\kappa}$  and that  $\frac{1-S_1^*(t)}{S_1^*(t)} = (t\phi\rho)^\kappa$
- Likewise,  $\frac{1-S_2^*(t)}{S_2^*(t)} = (t\rho)^\kappa \Rightarrow$  odds ratio is  $\phi^\kappa$
- Interpretation: among responders, the odds of responding prior to time  $t$ , is  $\phi^\kappa$  times greater for trt 1 than for trt 2



# Nonparametric estimation of $\hat{S}^*(t)$

$$\hat{S}_1^*(t) = \frac{\hat{S}_1(t) - (1 - \hat{p}_1)}{\hat{p}_1} \text{ where } \hat{S}_1, \hat{p}_1 \text{ are obtained from KM methods}$$

KM Estimates of  $S(t)$  and  $S^*(t)$  for one Treatment



## Nonparametric tests for $S_1^* = S_2^*$

- If  $p_1 = p_2$ , then a test of  $S_1 = S_2$  is a test of  $S_1^* = S_2^*$ 
  - since  $S_1^* = S_2^*$  if and only if  $S_1 = S_2$
- If assume  $p_1 = p_2$  (perhaps, after testing) test  $S_1 = S_2$  with logrank test
- But logrank test of  $S_1^* = S_2^*$  (when  $p_1 = p_2$ ) it will be highly inefficient (e.g., power of .2 when more appropriate methods will have a power of .9)
- Why? Log-rank test is most powerful in the case of "proportional hazards". But in a mixture of responders and nonresponders in which  $p_1 = p_2$ , this condition is violated big time
- When  $p_1 = p_2$  Orazem (Ph.D. Thesis, Columbia University, 1991) proposed a weighted log-rank test under the assumption of proportional conditional hazards

- When  $p_1 = p_2$ , the Orazem's weighted log-rank test is

$$T_{WLR} = \frac{\sum w(t_k) \{d_{1k} - E_{1k}\}}{\sqrt{\sum w^2(t_k) \text{Var}(d_{1k})}} \quad \text{where } w(t_k) = 1 - \Lambda_1^*(t) \frac{S_1(u)}{S_1(t)}$$

- $w(t_k)$ 's are optimal weights when  $\frac{h_2^*(t)}{h_1^*(t)} = \psi$  for all  $t \in (0, u)$
- under the null, an estimator of  $w(t_k)$  is  $\hat{w}_{t_k} = 1 - \frac{\hat{S}(u)}{\hat{S}(t_k)} \sum_{i=1}^k (d_i/n^*(t_i))$
- $n^*(t_i) = \sum_{t_k \geq t_i} \{c_k + (1-c_k)(1 - \frac{\hat{S}(u)}{\hat{S}(t_k)})\}$  is the est # of responders at "risk" at  $t_i$
- numerator in  $T_{WLR}$  can be shown to be equal  $\sum_k \{d_{1k} - d_k \frac{n_1^*(t_k)}{n^*(t_k)}\}$
- Under the null, with no ties,  $T_{WLR}$  converges in distrib. to  $N(0,1)$

## Tamura, et.al (Statistics in Medicine, 2000)

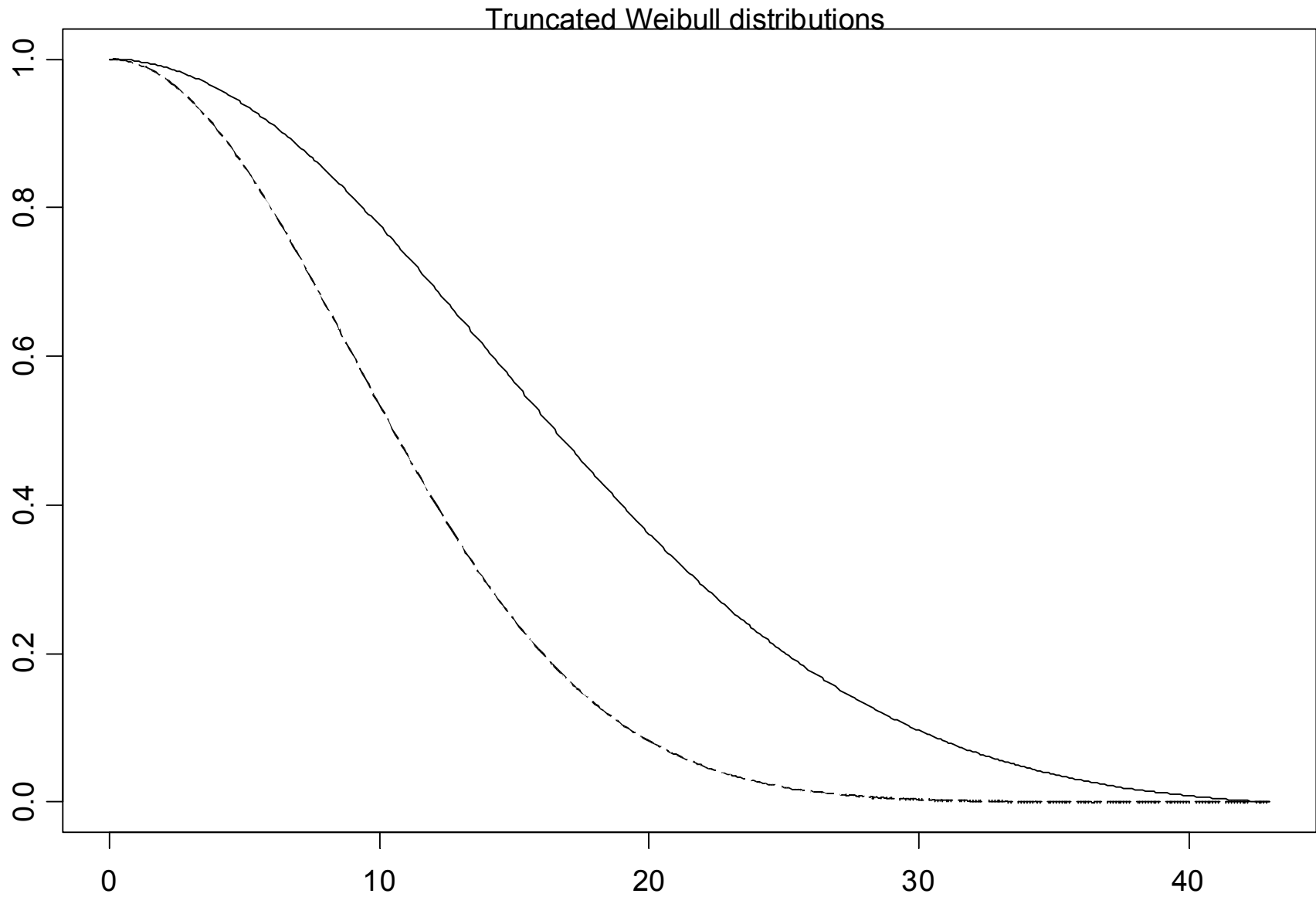
- Use a Cramer-von Mises test statistic to test  $S_1^* = S_2^*$
- $$W^2 = -(n_1 \hat{p}_1)(n_2 \hat{p}_2) / (n_1 \hat{p}_1 + n_2 \hat{p}_2) \int [\hat{S}_1^*(t) - \hat{S}_2^*(t)]^2 d\hat{S}^*(t)$$
- where  $\hat{S}^*(t) = \sum n_i \hat{p}_i \hat{S}_i^*(t) / \sum n_i \hat{p}_i$  is the est. of the common  $S^*(t)$
- Tamura, et. al derived the asymptotic distribution - holds with ties
- Hard to tabulate percentiles
- So they proposed a Bootstrap procedure in order to obtain an empirical distribution of  $W^2$  under the null.
- For group 1, for example, resample from  $B(\hat{p}_1), \hat{G}_1, \hat{S}^*$
- If the  $W^2$  equals the  $q$ th percentile of the empirical distribution, then the p-value =  $\frac{100-q}{100}$

- Tamura et. al conducted simulations to evaluate their procedure. I simulated data from the exact same distributions as Tamura.

The simulation specifications are:

- Interval censoring case: visits at days 5,10,15,22,29,36,43
- Set  $p_1 = p_2 = .6$
- For group 1,  $T \sim \text{Weibull}(\lambda = 1/400, \gamma = 2)$  truncated at day 43  
 $\Rightarrow$  median=16.6 and 90% respond within 30 days
- Group 2:  $S_2^*(t) = \{S_1^*(t)\}^\beta$   $\beta=1$  or 2.5; when  $\beta=2.5$ , median=10.4
- Censoring distribution (i.e., time to dropout), also truncated Weibull
  - $U \sim W_{tr}(\lambda = 1/40^4, \gamma = 4) \Rightarrow \approx 35\%$  censored prior to day 43
- Take  $\min(t,u)$  and round up to nearest scheduled visit day
- Used a sample size of 75 per group, and 1000 iterations

# The $S^*(t)$ 's from which data was simulated



Time to onset data  $\sim W_{tr} med_1 = 16.6$ , in alternative  $med_2 = 10.4$

Time to censoring  $\sim W_{tr} \approx 35\%$  dropout rate

$p_1 = .6, p_2 = .6, n = 75/\text{group}, 1000$  iterations

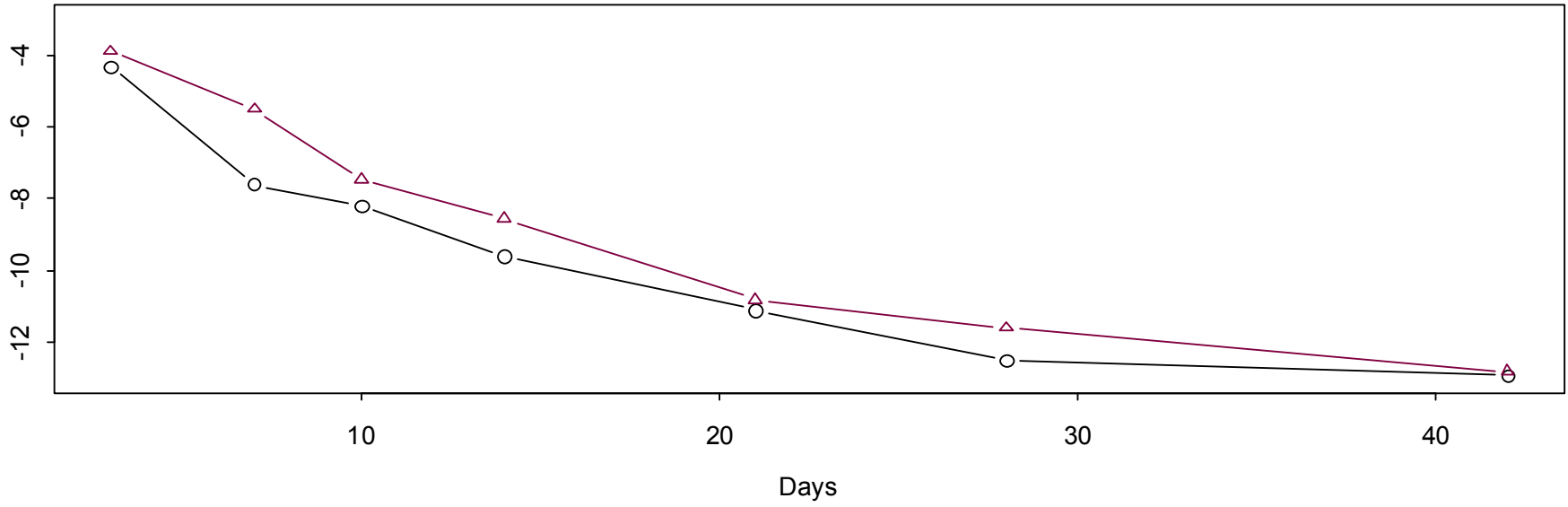
Method	Rejection rate	
	Null	Alternative
Cramer-von Mises	.048	.904
$T_{WLR}$	.052	.911
M-M Weibull	.056	.929
M-M Log logistic	.059	.875
log-rank test	.045	.256

## Exploratory analysis of a real dataset

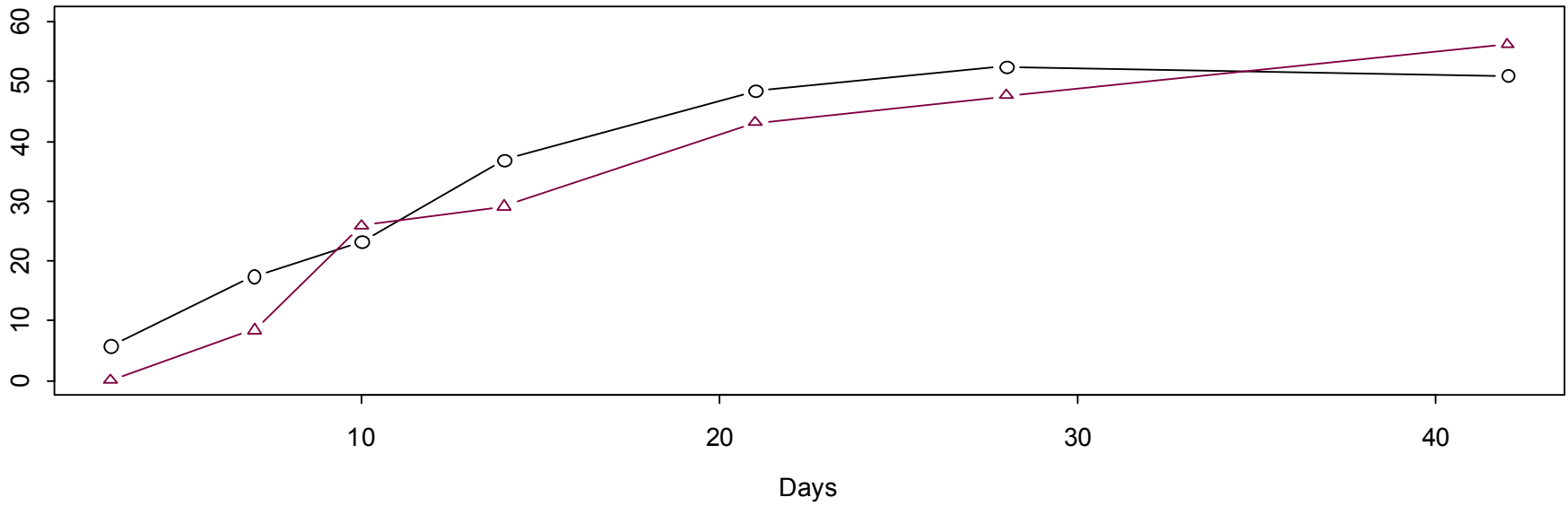
- 6 week MDD study with 5 groups, 3 doses of test drug, PBO, and a positive control
- comparison in this illustration is between the positive control (N=76, 51 completers) and one dose of the test drug (N=79, 56 completers)
- Time to  $\geq 50$  reduction from baseline in HAM-D total
  - partially sustained ( $\geq 35$  reduction from baseline) at all subsequent visits
  - early terminators were classified as responders if they had a  $\geq 50$  reduction from baseline at last two visits
  - Visits beyond 2 days of last day of medication were not considered
  - Visits  $> 38$  days were set equal to 42 days



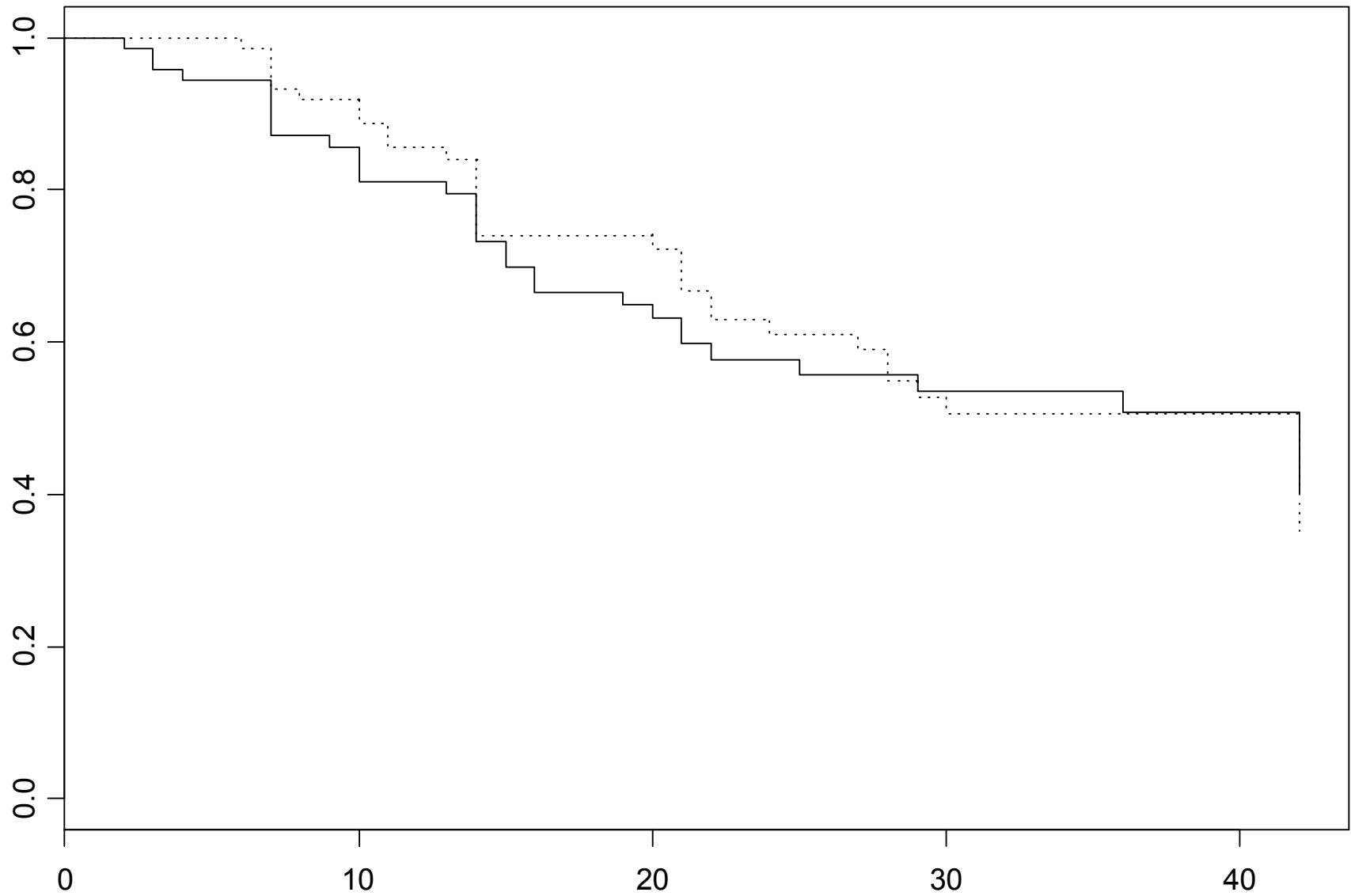
Mean change from BL



% of subjects responding



KM Estimates of  $S(t)$ : logrank test  $p=0.89$



Estimates of  $S^*(t)$  weighted logrank test:  $p=.33$

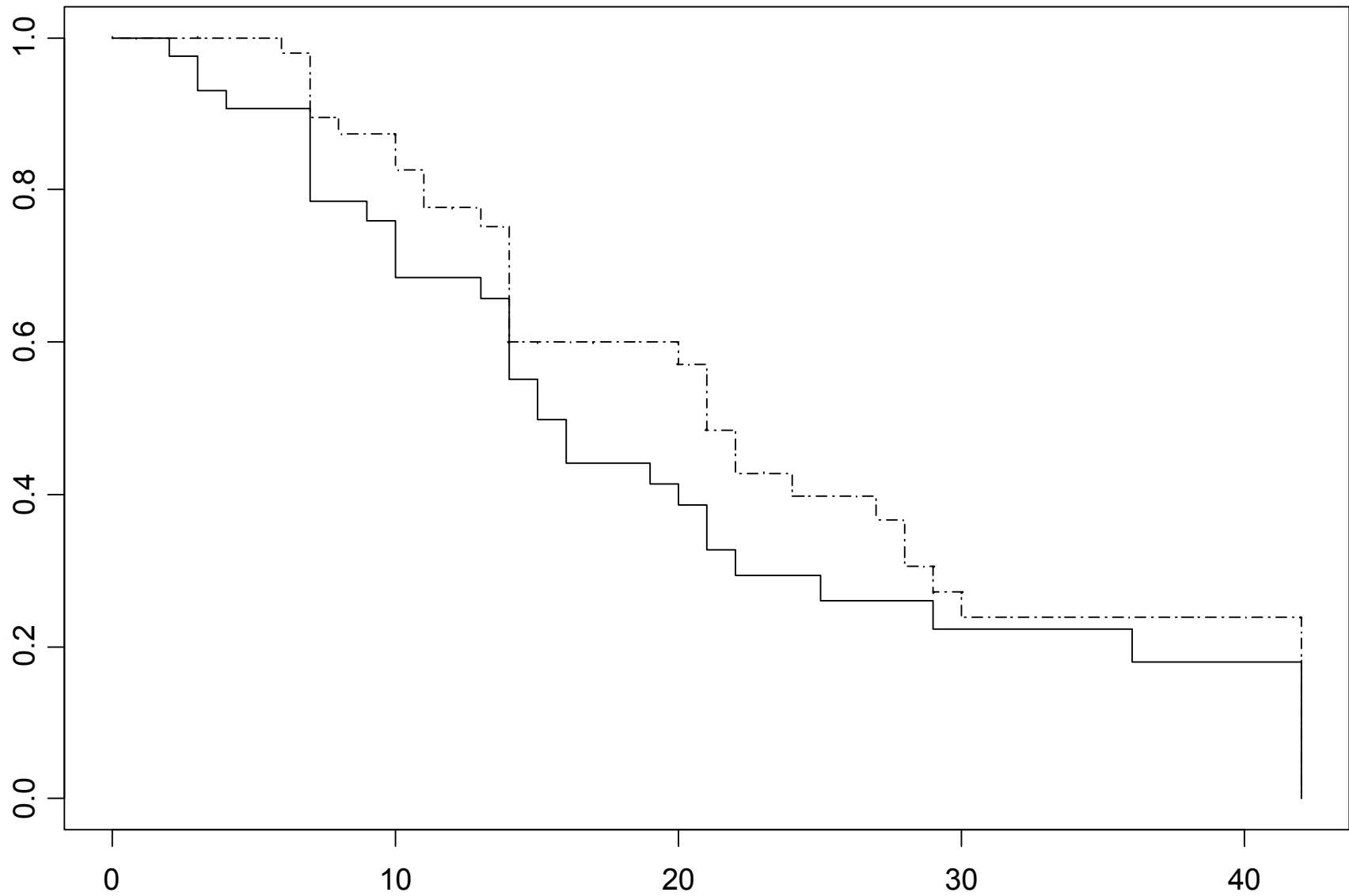
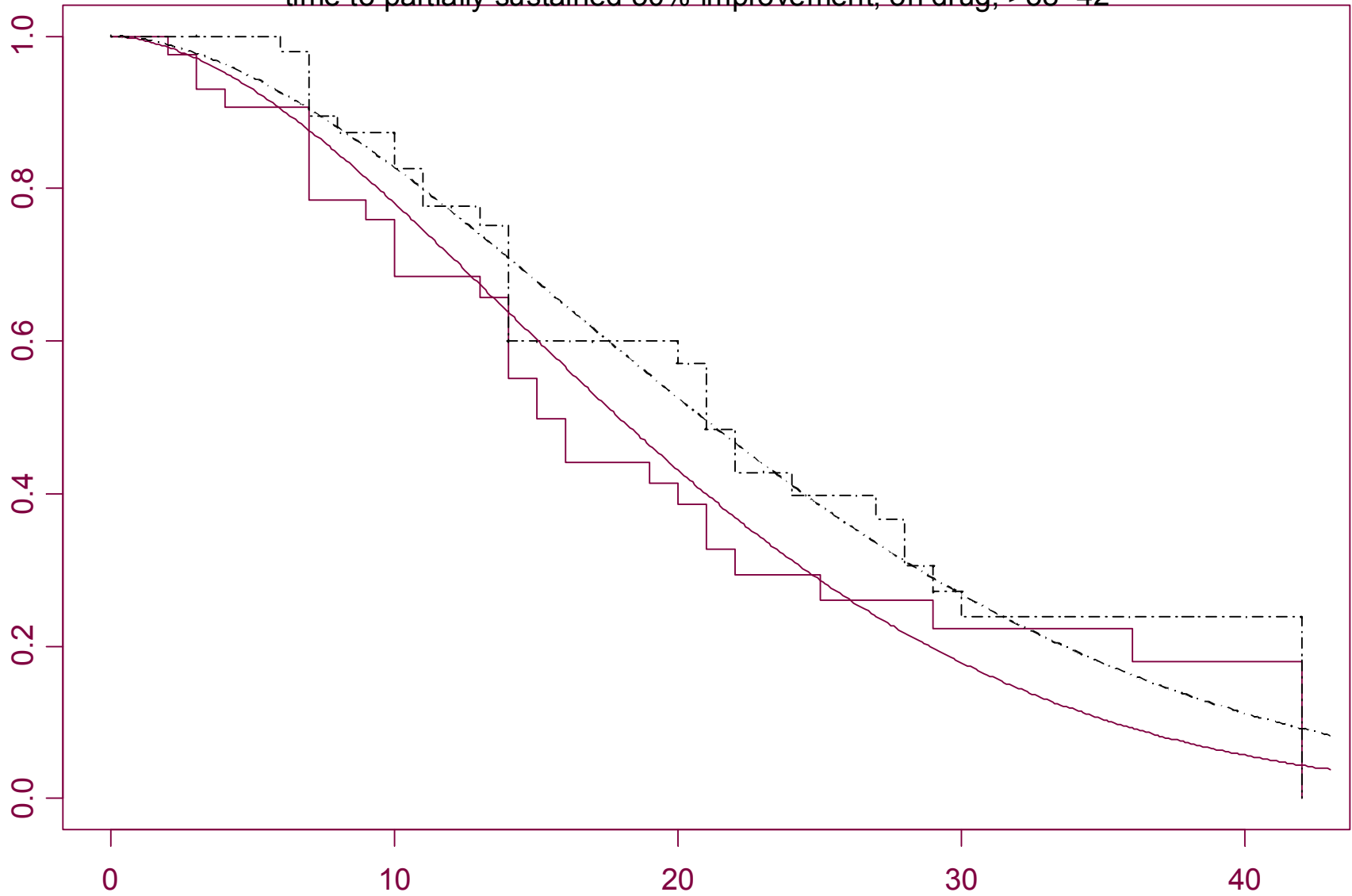


Fig. 4

# KM estimates of $S^*(t)$ 's along with fitted Weibull models

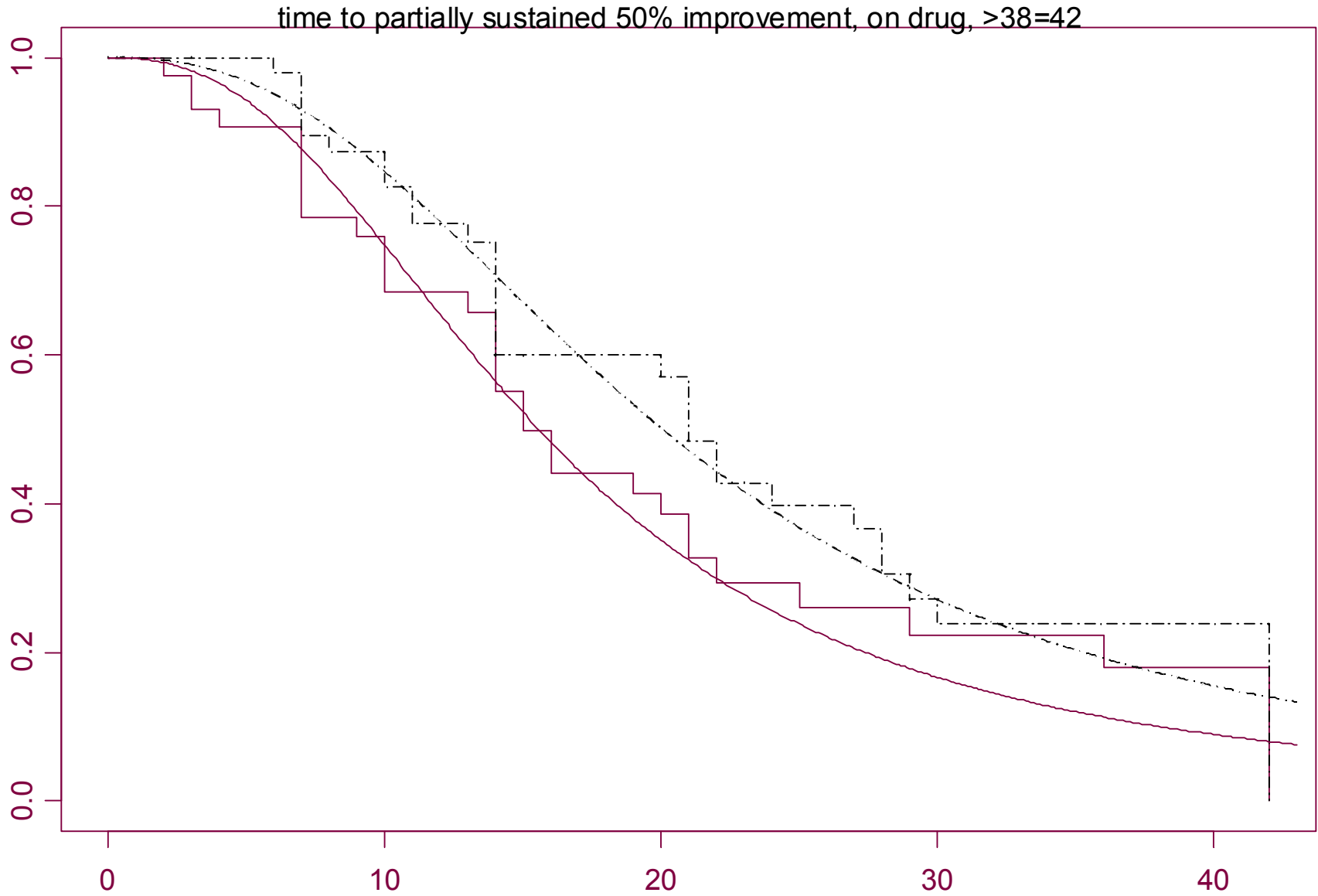
time to partially sustained 50% improvement, on drug, >38=42



mixed modeling allowing different p's test of  $s_i=1$ :  $p=.29$

Fig. 5

KM estimates of  $S^*(t)$ 's along with fitted log logistic models



mixed modeling allowing different p's test of  $s_i=1$ :  $p=.14$

## Assessing Model Fit

Specifically, assess the appropriateness of the chosen time to event distribution

- Plot log of the KM estimated conditional cumulative hazard vs log time
- Plot KM estimated log-odds of response beyond  $t$  vs. log time
- Likelihood ratio test of the PH or PO assumption
- Comparing the KM-type estimates of the conditional hazard to those estimated from the parametric models
- Assess whether the Cox-Snell residuals behave as if they are a censored sample from unit exponential distribution

- Recall that for the Weibull model,  $S_2^*(t) = e^{-\lambda t^\gamma}$   
 $\Rightarrow \log\{-\log S_2^*(t)\} = \log \lambda + \gamma \log t$
- Substitute the KM estimate  $\widehat{S}_2^*(t)$  for  $S_2^*(t)$  in the above equation
- If a Weibull distribution is tenable, then  $\widehat{S}_2^*(t)$  should be "close" to  $S_2^*(t)$  and
- a plot of  $\log\{-\log \widehat{S}_2^*(t)\}$  of  $\log t$  should yield an approximately straight line
- As shown earlier, for the loglogistic model,  $\frac{1-S_2^*(t)}{S_2^*(t)} = (t\rho)^\kappa$   
 $\Rightarrow \log \frac{S_2^*(t)}{1-S_2^*(t)} = -\kappa \log \rho - \kappa \log t$
- a plot of  $\log \frac{\widehat{S}_2^*(t)}{1-\widehat{S}_2^*(t)}$  of  $\log t$  should yield an approximately straight line

- In the Weibull model, the assumption of PH corresponds to the assumption of a common shape parameter  $\gamma$
- Let each the conditional survivor function for each treatment group have a separate scale **and** shape parameter, i.e.,  $S_i^*(t) = e^{-\lambda_i t^{\gamma_i}}$
- compare the loglikelihood of this model with the PH model which imposes the constraint of a common shape parameter  $\gamma$
- Analogously, the assumption of PO in the loglogistic model corresponds to the assumption of a common shape parameter  $\kappa$
- Likelihood ratio test results:
  - Test of PH in Weibull PH model:  $p=.18$
  - Test of PO in loglogistic PO model:  $p=.23$



- For a particular group or sample of survival data, the KM type estimator of the hazard in the interval  $t_k$  to  $t_{k+1}$  is  $\frac{d_k}{n_k \Delta_k}$  where  $d_k$  is the number of "deaths" at the  $k^{th}$  death time,  $n_k$  is the number at "risk" at time  $t_k$ , and  $\Delta_k = t_{k+1} - t_k$
- Let's denote the KM estimator of  $S(t)$  in the interval  $[t_k, t_{k+1})$  as  $\hat{S}(t_{\Delta_k}) = \prod_{j=1}^k (1 - \frac{d_j}{n_j})$   

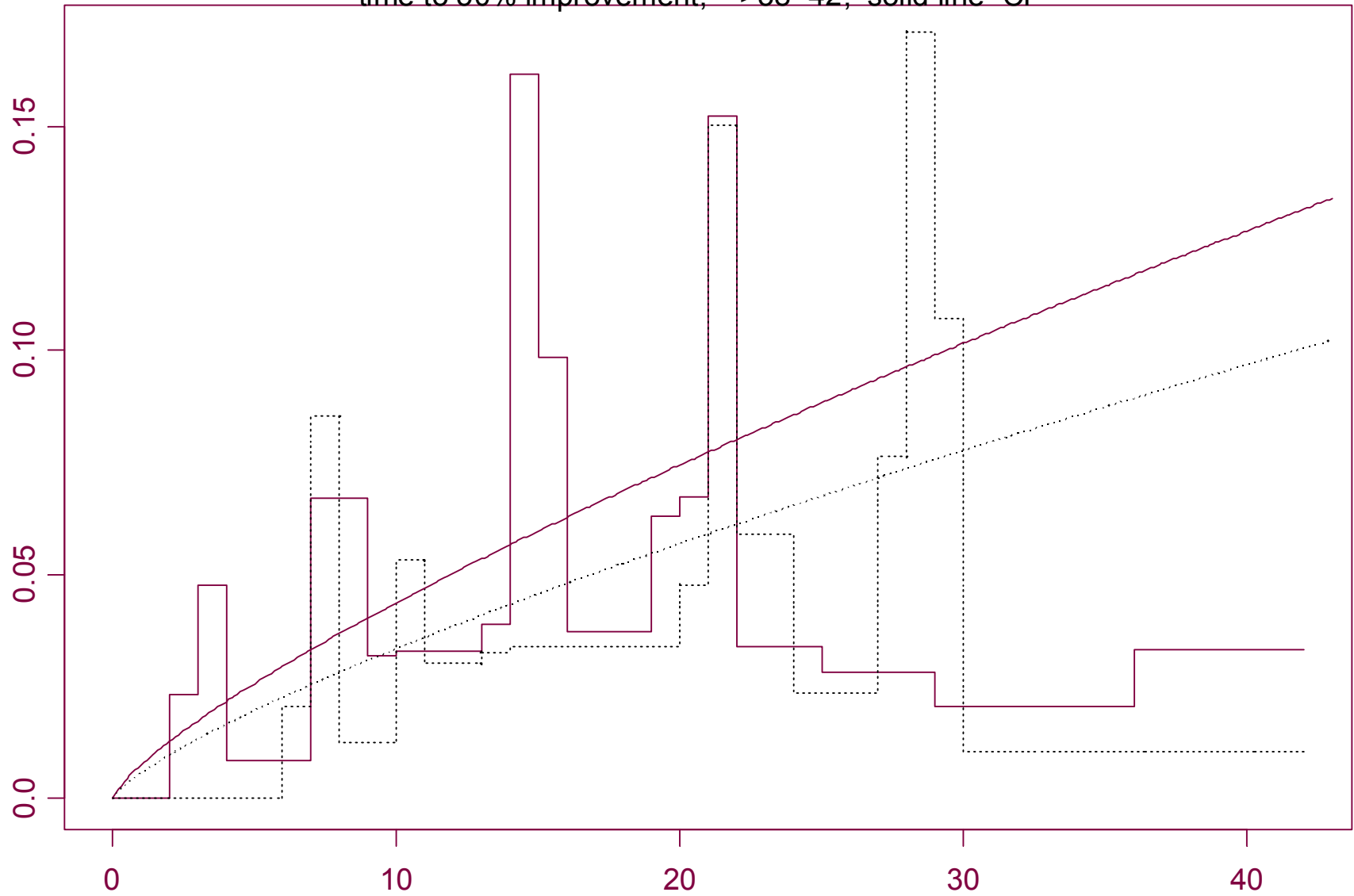
$$\Rightarrow \hat{S}(t_{\Delta_k}) = \hat{S}(t_{\Delta_{k-1}}) (1 - \frac{d_k}{n_k}) \Rightarrow \frac{d_k}{n_k} = \frac{\hat{S}(t_{\Delta_{k-1}}) - \hat{S}(t_{\Delta_k})}{\hat{S}(t_{\Delta_{k-1}})}$$
- It then follows that a corresponding estimator of the conditional hazard in the interval  $t_k$  to  $t_{k+1}$  is

$$\frac{\hat{S}^*(t_{\Delta_{k-1}}) - \hat{S}^*(t_{\Delta_k})}{\hat{S}^*(t_{\Delta_{k-1}}) \Delta_k}$$

- Compare these KM type conditional hazard estimates to those estimates obtained from the Weibull and loglogistic models

# KM-type estimates of $h^*(t)$ 's

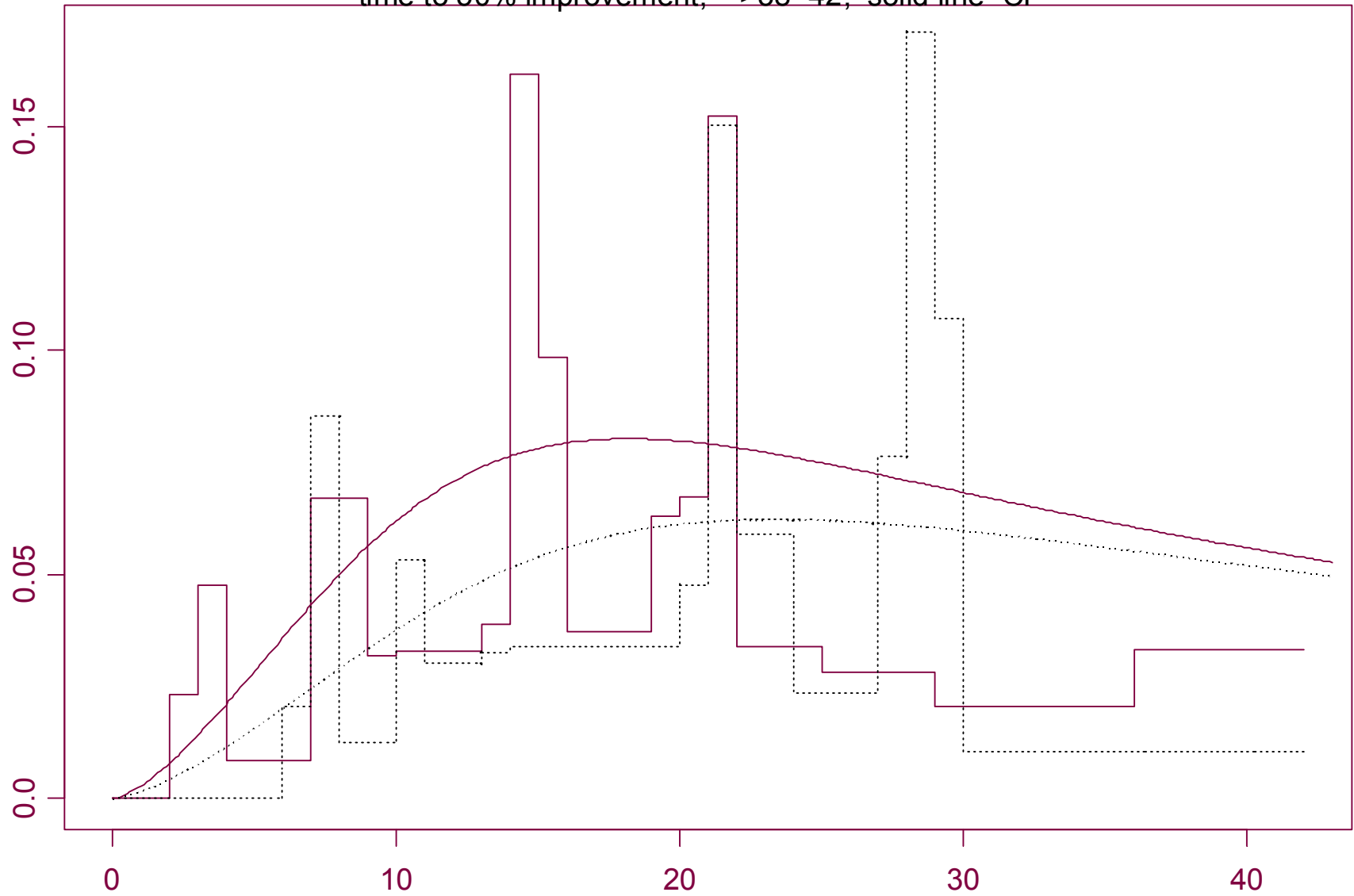
time to 50% improvement,  $>38=42$ , solid line=CP



with estimated hazards based on Weibull model

# KM-type estimates of $h^*(t)$ 's

time to 50% improvement,  $>38=42$ , solid line=CP



with estimated hazards based on loglogistic model

Cox-Snell residuals:  $r_{Ci} = -\log \hat{S}_i(t_i)$

$t_i$  is the observed survival time for subject  $i$

In general if  $T \sim S(t)$ , then  $Y = -\log S(T) \sim \exp(1)$

proof:  $f_Y(y) = f_T(t) / \left| \frac{dy}{dt} \right| = f_T(t) / \frac{f_T(t)}{S(t)} = S[S^{-1}(e^{-y})] = e^{-y}$

$\Rightarrow$  if model fits well,  $r_{Ci}$  will represent a censored sample  $\sim \exp(1)$

Obtain  $r_{Ci}$ , treat as survival times and compute KM estimates  $\hat{S}(r_{Ci})$ ; residuals for censored times are considered censored observations

If  $\log\{-\log \hat{S}(r_{Ci})\}$  vs  $\log r_{Ci}$  yields a reasonably straight line with unit slope and zero intercept model fits well

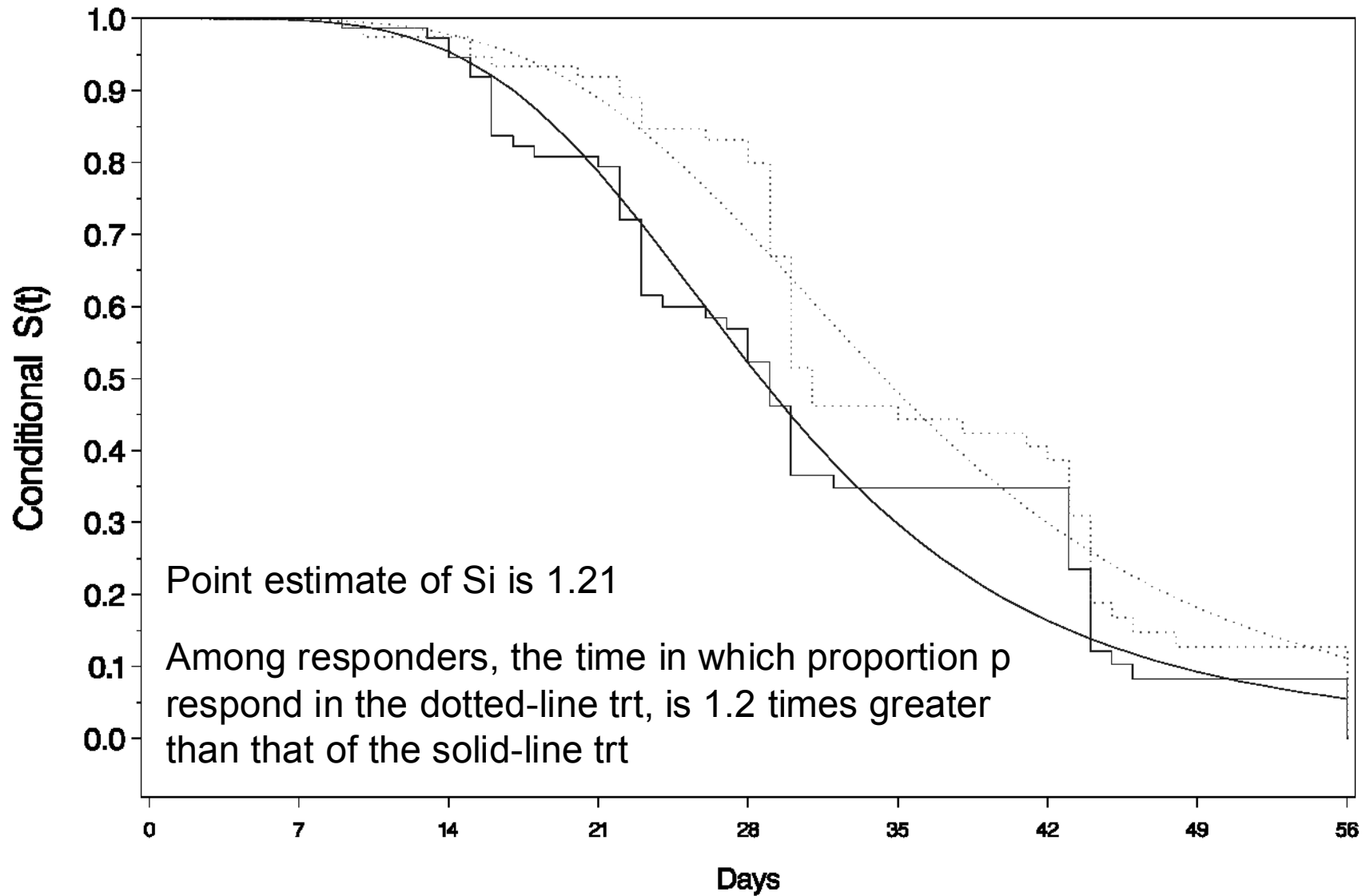
Can one apply this residual analysis in this mixture model setting?

In the case of 2 treatments, the Cox-Snell residual for the  $i^{th}$  subject in the  $g^{th}$  group is:  $r_{Ci} = -\log\{\hat{S}_g^*(t_i)\hat{p}_g + (1 - \hat{p}_g)\}$

## A follow-up example in which the loglogistic parametric model as specified a priori

- 8 week MDD study with 3 groups, combo of test drug + positive control (PC), PC alone, and PBO
- comparison is between combo (N=103, 72 completers) and PC (N=101, 74 completers)
- Time to  $\geq 50$  reduction from baseline in MADRS total
  - must be sustained at all subsequent visits
  - early terminators were classified as responders if they had a  $\geq 50$  reduction from baseline at last two visits
  - Visits between 52 and 59 days were set equal to 56 days

Test of  $S_i=0$ , yielded a  $p = 0.015$ ; logrank test,  $p=0.28$



## Issues, concerns and criticisms of this mixture method/paradigm

- discard information by dichotomizing a nice “continuous” variable
- how does one dichotomize (onset, response, etc.)?
- sustained response or partially sustained, and for how long?
- interval censoring (i.e., unlike many applications of survival analysis, don’t know precise time of response—only that it was between visits)
- time to response and time to censoring may not be independent
- $S_1^* < S_2^*$  only meaningful if  $p_1 \geq p_2$  (perhaps requiring  $\hat{p}_1 \geq \hat{p}_2$ )?
- most applicable for comparing 2 efficacious treatments
- estimates of  $p$  and hence of  $S^*$  can be unstable
- parametric models may not be robust to departures from assumptions
- no standard ready-to-use software procedures
- analyze time to response among responders to obtain  $\hat{S}_i^*$  and for testing
  - works reasonably well, but
  - 1) subgroup analysis; not ITT, 2) censored subjects provide info